SM2 10.3: Probability, Independence, & the Addition Rule

You have an equally likely chance of randomly choosing any integer between 1 and 50. Find the probability of each of the following events.

1) Choosing an even number

25 of the numbers are even

$\frac{25}{50} = \frac{1}{2} = .5$

2) Choosing a perfect square

Perfect Squares: 1, 4, 9, 16, 25, 36, 49

$\frac{7}{50} = .14$

3) Choosing a factor of 150

Factors of 150: 1, 2, 3, 5, 6, 10, 15, 25, 30, 50

$\frac{10}{50} = \frac{1}{5} = .2$

4) Choosing a two digit number

Two digit numbers: from 10 to 50 (41 numbers)

$\frac{41}{50} = .82$

5) Choosing a multiple of 4

Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48

$\frac{12}{50} = \frac{6}{25} = .24$

6) Choosing a number less than but not equal to 35

There are 34 numbers less than but not equal to 35

$\frac{34}{50} = \frac{17}{25} = .68$

7) Choosing a prime number

Prime numbers: 2,3,5,7,11,13,17,19,23,29,31,37,41,43,47

$\frac{15}{50} = \frac{3}{10} = .3$

8) Choosing a perfect cube

Perfect Cubes: 1, 8, 27

$\frac{3}{50} = .06$

You randomly draw two marbles out of a bag, <u>replacing the first marble</u> before drawing the second marble. The bag has 10 black, 8 red, 4 white and 6 blue marbles. Find the probability of each of the following events:

9) A white marble, then a red marble is selected

$P(W) \cdot P(R) = \frac{4}{28} \cdot \frac{8}{28} = \frac{32}{784} = \frac{2}{49} \approx .041$

10) A red marble is NOT selected, then a blue marble

$P(R^C) \cdot P(Bl) = \frac{20}{28} \cdot \frac{6}{28} = \frac{120}{784} = \frac{15}{98} \approx .153$

11) A green marble, then a green marble is selected

$P(G) \cdot P(G) = \frac{0}{28} \cdot \frac{0}{28} = \frac{0}{784} = 0$

Green is an impossible event

12) A blue or black marble is selected, then a white marble

$P(Bl\ or\ Bk) \cdot P(W) = \frac{16}{28} \cdot \frac{4}{28} = \frac{64}{784} = \frac{4}{49} \approx .082$

13) Selecting a marble that is NOT black, then a blue

$P(Bk^C) \cdot P(Bl) = \frac{18}{28} \cdot \frac{6}{28} = \frac{108}{784} = \frac{27}{196} \approx .138$

14) A green marble, then a blue marble is selected

$P(G) \cdot P(Bl) = \frac{0}{28} \cdot \frac{6}{28} = \frac{0}{784} = 0$

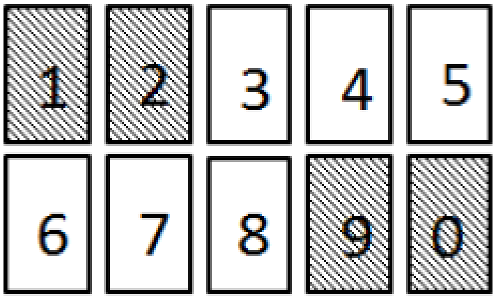
15) A yellow marble and then a red marble is drawn

$P(Y) \cdot P(R) = \frac{0}{28} \cdot \frac{8}{28} = \frac{0}{784} = 0$

16) A color that is NOT yellow is drawn both times

$P(W) \cdot P(R) = \frac{28}{28} \cdot \frac{28}{28} = \frac{784}{784} = 1$

Drawing a card from the cards on the left, determine the probability of each of the following events. Show the Addition Rule equation for each one.



Example: $P(Odd\ or\ Shaded) = \frac{5}{10} + \frac{4}{10} - \frac{2}{10} = \frac{7}{10}$

17) $P(Even\ or\ Shaded)$
$= .7$

18) $P(White\ or\ Odd)$
$= \frac{8}{10} = \frac{4}{5} = .8$

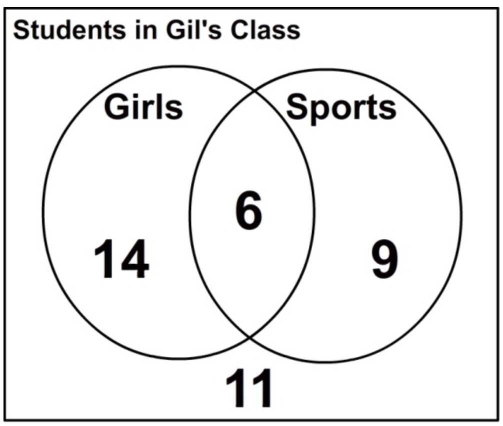
19) $P(Less\ than\ 4\ or\ shaded)$
$= \frac{5}{10} = \frac{1}{2} = .5$

20) $P(Greater\ than\ 5\ or\ shaded)$
$= \frac{7}{10} = .7$

21) $P(Factor\ of\ 10\ \ or\ white)$
$= \frac{8}{10} = \frac{4}{5} = .8$

22) $P(less\ than\ or\ equal\ to\ 2\ \ or\ shaded)$
$= \frac{4}{10} = \frac{2}{5} = .4$

Use the Venn diagram to find the probabilities of the given events. Give your probabilities as reduced fractions.



23) $P(Girl)$
$= \frac{20}{40} = \frac{1}{2} = .5$

24) $P(Boy)$
$P(not\ a\ Girl) = 1 - P(Girl) = 1 - \frac{1}{2} = \frac{1}{2} = .5$

25) $P(Student\ is\ from\ Gil's\ class)$
All the students are from Gil's class so the probability is 1

26) $P(Girl\ and\ Plays\ Sports)$
$= \frac{6}{40} = \frac{3}{20} = .15$

27) $P(Girl\ or\ Plays\ Sports)$
$= \frac{29}{40} = .725$

28) $(is\ not\ Girl\ and\ does\ not\ Play\ Sports)$
$= \frac{11}{40} = .275$

29) Are the events “being a girl” and “playing a sport” independent events? Show the work that supports your answer.

If they are independent, then
$P(girl\ and\ plays\ a\ sport) = P(girl) \cdot P(plays\ a\ sport)$
$P(girl\ and\ plays\ a\ sport) = \frac{6}{40} = \frac{3}{20}$

$P(girl) \cdot P(plays\ a\ sport) = \frac{20}{40} \cdot \frac{15}{40} = \frac{3}{16}$

$\frac{3}{20} \neq \frac{3}{16}$ so they are <u>not</u> independent

Find the marginal totals for the table. Then use the table to find the probabilities for each event.

There are 200 people whose gender and hair color have been counted

	Brown	Blonde	Red	Black	Other	Total
Male	42	11	11	13	23	100
Female	47	16	10	13	14	100
Total	89	27	21	26	37	200

30) $P(male)$

$\frac{100}{200} = \frac{1}{2} = .5$

31) $P(red)$

$\frac{21}{200} = .105$

32) $P(other)$

$\frac{37}{200} = .185$

33) $P(blonde \cap male\ )$

$\frac{11}{200} = .055$

34) $P(black\ and\ female)$

$\frac{13}{200} = .065$

35) $P(brown \cap not\ male)$

$\frac{47}{200} = .235$

36) $P(red\ and\ black)$

Cannot happen at the same time:

$\frac{0}{200} = 0$

37) $P(female \cap not\ other)$

$\frac{86}{200} = \frac{43}{100} = .43$

38) Are *being male* and *having blonde hair* independent events? Show your work.

$$Does: P(M\ and\ B) = P(M) \cdot P(B)\ ?$$

$$P(M\ and\ B) = \frac{11}{200} = \frac{22}{400}$$

$$P(M) \cdot P(B) = \frac{100}{200} \cdot \frac{27}{200} = \frac{27}{400}$$

$$\frac{22}{400} \neq \frac{27}{400}$$

NOT INDEPENDENT

39) Are *being female* and *having black hair* independent events? Show your work.

$$Does: P(F\ and\ Bk) = P(F) \cdot P(Bk)\ ?$$

$$P(F\ and\ Bk) = \frac{13}{200}$$

$$P(F) \cdot P(Bk) = \frac{100}{200} \cdot \frac{26}{200} = \frac{26}{400} = \frac{13}{200}$$

$$\frac{13}{200} = \frac{13}{200}$$

YES, THEY ARE INDEPENDENT

40) A photographer has taken 8 black and white photographs and 10 color photographs for a brochure. If 4 photographs are selected at random without replacing them, find the following probabilities.

a. What is the probability of picking first 2 black and white photographs, then 2 color photographs?

$$\frac{8}{18}\frac{7}{17}\frac{10}{16}\frac{9}{15} = \frac{14}{204} \approx 0.068$$

b. What is the probability that all the photographs are all black and white?

$$\frac{8}{18}\frac{7}{17}\frac{6}{16}\frac{5}{15} = \frac{7}{306} \approx 0.023$$

c. What is the probability that all the photographs are color?

$$\frac{10}{18}\frac{9}{17}\frac{8}{16}\frac{7}{15} = \frac{14}{204} \approx 0.068$$

d. What is the probability that all 4 are the same type (all black and white or all color)?

Part b + Part c : $\frac{7}{306} + \frac{14}{204} = \frac{14}{153}$ $or\ 0.023 + 0.068 \approx 0.091$

41) There are 7 blue pens, 3 black pens, and 2 red pens in a drawer. If you select three pens at random with no replacement, find the following probabilities.

a. What is the probability that you will select a blue pen, then a black pen, then another blue pen?

$$\frac{7}{12}\frac{3}{11}\frac{6}{10} = \frac{21}{220} \approx 0.095$$

b. What is the probability that all three are red?

There are only 2 red pens, so 0

c. What is the probability that all three pens are the same color?

$$\frac{7}{12}\frac{6}{11}\frac{5}{10} + \frac{3}{12}\frac{2}{11}\frac{1}{10} + \frac{2}{12}\frac{1}{11}\frac{0}{10} = \frac{9}{55} \approx .164$$

42) Tammy's mom is baking cookies for a bake sale. When Tammy comes home, there are 22 chocolate chip cookies, 18 sugar cookies, and 15 oatmeal cookies on the counter. Tammy sneaks into the kitchen, grabs a cookie at random, and eats it. Five minutes later, she does the same thing with another cookie.

a. What is the probability that neither of the cookies was a chocolate chip cookie?

$$\frac{33}{55}\frac{32}{54} = \frac{16}{45} \approx .356$$

b. What is the probability that both cookies were the same flavor?

$$\frac{22}{55}\frac{21}{54} + \frac{18}{55}\frac{17}{54} + \frac{15}{55}\frac{14}{54} = \frac{163}{495} \approx .329$$

43) There are 2 Root Beers, 2 Sprites, 3 Mountain Dews, and 1 Gatorade left in the vending machine at school. The machines buttons are broken and will randomly give you a random drink no matter what button you push. Find the probability of each outcome.

a. $P(root\ beer\ then\ root\ beer)$

$$\frac{2}{8}\frac{1}{7} = \frac{1}{28} \approx .036$$

b. $P(root\ beer\ then\ mountain\ dew)$

$$\frac{2}{8}\frac{3}{7} = \frac{3}{28} \approx .107$$

c. $P(sprite\ then\ gatorade)$

$$\frac{2}{8}\frac{1}{7} = \frac{1}{28} \approx .036$$

d. $P(mountain\ dew\ then\ mountain\ dew\ then\ mountain\ dew)$

$$\frac{3}{8}\frac{2}{7}\frac{1}{6} = \frac{1}{56} \approx .018$$

Determine whether or not the following events are independent.

44) If $(A) = 0.7$, $P(B) = 0.3$ and $P(A \cap B) = 0.21$ are events A and B independent? Why or why not?

$$Yes, because\ P(A \cap B) = P(A) \cdot P(B)$$
$$0.21 = 0.7 \cdot 0.3$$

45) Janeen has a dozen cupcakes. Three are chocolate with white frosting, two are chocolate with yellow frosting, four are vanilla with white frosting and three are vanilla with yellow frosting.

a. Fill in the table below from the information above.

	White	Yellow	Total
Chocolate	3	2	5

Vanilla	4	3	7
Total	7	5	12

b. Are chocolate cake flavor and white frosting color independent?

$$\text{Does } P(\text{Chocolate and White}) = P(\text{Chocolate}) \cdot P(\text{White})\ ?$$

$$P(\text{Chocolate and White}) = \frac{3}{12} = \frac{36}{144}$$

$$P(\text{Chocolate}) \cdot P(\text{White}) = \frac{5}{12} \cdot \frac{7}{12} = \frac{35}{144}$$

$$\frac{36}{144} \neq \frac{35}{144} \text{ so they are NOT independent}$$